Topic- A Mathematical Model on the two phase renal diastolic blood flow in arterioles with special reference to Diabetes.

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Abstract - In the present paper we have formulated the renal blood flow in arterioles. Keeping in view the nature of renal circulatory system in human body. The viscosity increases in the arterioles due to formation of Rolex along axis by red blood cells, as we know the arterioles are remote from heart and proximate to the kidney. P.N. Pandey and V. Upadhyay have considered the blood flow has two phased , one of which is that of red blood cells and other is Plasma .They have also applied the Herschel Bulkley non –Newtonian Model in bio-fluid mechanical set-up .We have collected a clinical data in case of Diabetes for Hematocrit v/s Blood Pressure. The graphical presentation for particular parametric value is much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical. The role of Hematocrit is explicit in the determination of blood pressure in case of renal disease –Diabetes.

(1) Introduction (Description of Bio-Physical problem) –

Kidney is a very important organ. They clean the blood & regulate the fluid in the body. Kidneys are vulnerable to many diseases. Some kidney diseases are life threatening and others can lead to complete kidney failure and require dialysis

Kidneys in the abdominal cavity, there are two, one on each side of the spine. have a bean-shaped structure, each kidney has a convex and concave surface. In human the kidneys are located

Blood is a complex fluid consisting of particulate solids suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base. 50% of the plasma and 45% of the blood cells and in 45% of the blood is RBCs and there is a few parts of the other cells. Which are ignorable, so one phase of the blood is plasma and 2^{nd} phase of the blood is RBCs.

Two phase renal blood flow is a study of measuring the blood pressure if hemoglobin known. The percentage of volume covered by blood cells in the whole blood is called hematocrit. This work will focus on two phase renal blood flow in arterioles with special reference to Diabetes.

A lot of work is available, but P.N.Pandey and V. Upadhyay (2001) discussed a some phenomena in two phase blood flow gave an idea on the two phase renal blood flow in arterioles with a renal disease Diabetes. The work of P.N. Pandey and V.Upadhyay in whole circulatory system but this work will focus on renal circulatory system, and renal circulatory system is a sub system of whole circulatory system. In this work, applied the Herschel Bulkley non-Newtonian model.

Here renal blood flow means blood flow in kidney tissue. We present an improvement on the previous work in the field and this is discussed separately below. The ultimate use of this model is to predict normal reference levels of two phase blood flow in arterioles for individual patients undergoing to Diabetes disease. According to David A. Farman, BS, and Stephanie C. Wu, DPM, MSc Belongs to the family of Diabetes called Diabetology. The prevalence of diabetes is increasing rapidly and is expected to reach epidemic proportion over the next decade. Recent research estimates that the number of people diagnosed with diabetes will rise from 23.7 million to 44.1 million between 2009 and 2034.[1] The Centers for Disease Control and

Prevention (CDC) further predict that up to one-third of U.S. adults could have diabetes by 2050 if Americans continue to gain weight and avoid exercise.

According to recent estimates, approximately 285 million people worldwide (6.6%) in the 20–79 year age group will have diabetes in 2010 and by 2030, 438 million people (7.8%) of the adult population, is expected to have diabetes.(1) The largest increases will take place in the regions dominated by developing economies.

In the Kidneys, filtration, re-absorption and secretion of the blood and removal of harmful chemicals called toxins. Renal circulation is the process in which blood mixed with toxins (harmful chemicals) flows to the renal artery in kidney, then to afferent arterioles then to glomerular capillary blood vessels, where waste products and excess water pass out of the blood stream, then to renal veins and finally to the heart

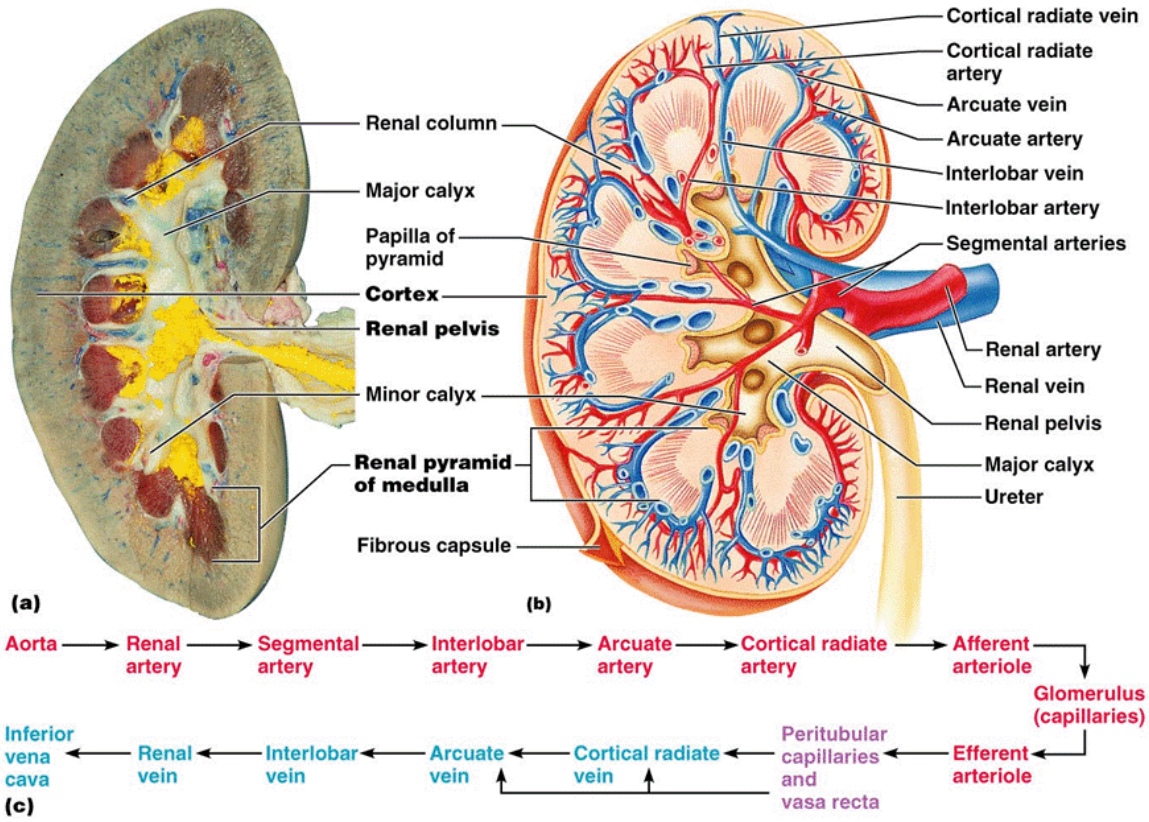


Fig:1

According to A.C. Guyton Renal circulation occurs due to blood pressure gradient, the high pressure of glomerular capillaries about 60 mm Hg causes rapid fluid filtration, whereas much lower hydrostatic pressure in the pertibular capillaries about 13 mm Hg permits rapid re-absorption. Renal blood flow is not a linear function of the blood pressure gradient.

(2) Basic Bio-fluid equation for two phase blood flow-

Let us the problem of blood flow in renal circulatory system is different from the problems in cylindrical tube and select generalized three dimensional orthogonal curvilinear coordinate system. Briefly described as E^3 called as Euclidean space. According to mishra the biophysical laws thus expressed fully hold good in any co-ordinate system which is a compulsion for the truthfulness of the laws (1990).

According to Sherman I.W. and Sherman V.,G Blood is mixed fluid. Mainly there are two phases in blood. The first phase is plasma, while the other phase is that of blood cells are enclosed with a semi-permeable membrance whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma. Thus, blood can be considered as a homogeneous mixture of two phases (1989).

(2.1) Equation of Continuity for two phase blood flow-

According to Singh P. and Upadhyay K.S. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , this X is replaced by H/100, where H is the Hematocrit the volume percentage of blood cells .Then the volume portion covered by the plasma will be 1-X . If the mass ratio of blood cells to plasma is r then clearly

$$r = \frac{X\rho_c}{(1-X)\rho_p} \tag{2.1}$$

where ϱ_c and ϱ_p are densities of blood cells and blood blood plasma respectively .Usually this mass ratio is not a constant , even then this may be supposed to constant in present context (1986)

The both phase of blood, i.e. blood cells and plasma move with the common velocity .Campbell and Pitcher has presented a model for two phase of blood separately (1958). Hence equation of continuity for two phases according to the principle of conservation of mass defined by J.N and Gupta R.C. as follow

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i)_{,i} = 0 \tag{2.2}$$

and $$\frac{\partial(1-X)\rho_p}{\partial z} + ((1-X)\rho_p v^i)_{,i} = 0 \tag{2.3}$$

Where, V is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood ρ_m as follow $\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p}$ (2.4)

Then equation (2.2) and (2.3) can be combined together as follow,

$$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,i} = 0 \tag{2.5}$$

(2.2) Equation of Motion for two phase blood flow-

According to Ruch, T.C. and H.D. The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973). Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^j)v^i_{,j} = -Xp_{,j}g^{ij} + X\eta_c(g^{jk}v^i_{,k})_{,j} \tag{2.6}$$

Similarly, taking the viscosity coefficient of plasma to be. The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \left\{(1-X)\rho_p v^i\right\} v^i_{,j} = -(1-X)p_{,j}g^{ij} + (1-X)\eta_c\left(g^{jk}v^i_{,k}\right)_{,j} \tag{2.7}$$

Now adding equation (2.6) and (2.7) and using relation (2.4), the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + \left(\rho_m v^j\right) v^i{}_{,j} = -p_{,j} + \eta_m \left(g^{jk} v^i{}_{,k}\right)_{,j} \qquad (2.8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

(3) Mathematical Modeling-

As the velocity of Blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law holds good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$T' = \eta_m e^n + T_p \left(T' \geq T_p\right)$ and $e = 0\,(T' < T_p)$ where, T_p is the yield stress.

When strain rate $e = 0\left(T' < T_p\right)$ a core region is formed which flows just like a plug. Let the radius of the plug be r_p. The stress acting on the surface of plug will be T_p. Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p$$

$$\Rightarrow r_p = 2\frac{T_p}{P} \qquad (3.1)$$

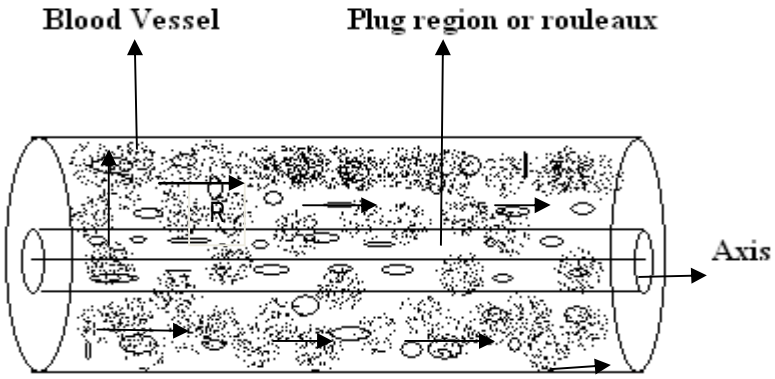


Fig (2): Herschel Bulkley blood flow

The Constitutive equation for test part of the blood vessel is

$T' = \eta_m e^n + T_p$ or $T' - T_p = \eta_m e^n = T_e$ Where, T_e = effective Stress

Whose generalized form will be as follows

$T^{ij} = -Pg^{ij} + T_e{}^{ij}$ Where, $T_e^{ij} = \eta_m \left(e^{ij}\right)^n$ While $e^{ij} = g^{jk} V_k^i$

Where , the symbols have their usual meanings.

Now we describe the basic equations for Herschel Bulkley blood flow as follows:

(3.1) Equation of Continuity-

$$\frac{1}{\sqrt{g\sqrt{(gV^i)_{,i}}}}=0$$

(3.2) Equation of Motion-

$$\rho_m \frac{\partial v^i}{\partial t}+\rho_m V^j V^i_{,j}=-T^{ij}_{e,j} \qquad (3.2)$$

Where all the symbols have their usual meanings, since, the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier: $X^1=r, X^2=\theta, X^3=Z$

Matrix of metric tensor in cylindrical co-ordinates is $\left[g_{ij}\right]$ and matrix of conjugate metric tensor is $\left[g^{ij}\right]$ whereas the chritoffel's symbols of 2nd kind are as follows:

$\left\{\begin{matrix}1\\2\ 2\end{matrix}\right\}=-r, \left\{\begin{matrix}1\\2\ 2\end{matrix}\right\}=\left\{\begin{matrix}1\\2\ 2\end{matrix}\right\}=\frac{1}{r}$ remaining others are zero.

The governing tensorial equations can be transformed into cylindrical forms which are follows: the equation of Continuity-: $\frac{\partial v}{\partial z}=0$

The equation of Motion-

r-component: $-\frac{\partial p}{\partial z}=0$, Θ-component: 0=0

z-component: $0=-\frac{\partial p}{\partial z}+\frac{\eta_m}{r}\left[r\left(\frac{\partial v_z}{\partial r}\right)^n\right]$

Here, this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r , v_z and p do not depend upon θ.

We get $v_z = v(r)$ and p=p(z) and $0=-\frac{dp}{dz}+\frac{\eta_m}{r}\left[r\left(\frac{dv}{dz}\right)^n\right]$ (3.3)

Since, pressure gradient $-\frac{dp}{dz} = P$

$r\left(\frac{dv}{dz}\right)^n=-\frac{pr^2}{2\eta_m}+A$, we apply boundary condition: at r=0. $V=V_0$ then A=0

$$\Rightarrow -\frac{dv}{dr}=\left(\frac{pr}{2\eta_m}\right)^{\frac{1}{n}}$$ Replace r from r-r_p

$$-\frac{dv}{dr}=\left(\frac{\frac{1}{2}pr-\frac{1}{2}pr_p}{\eta_m}\right)^{\frac{1}{n}} \Rightarrow \frac{dv}{dr}=-\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(r-r_p\right)^{\frac{1}{n}} \quad (3.4)$$

Integrating above equation (3.4) under the no slip boundary condition: v=0 at r=R so as to get:

$$V=\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\frac{n}{n+1}\left[\left(R-r_p\right)^{\frac{1}{n}+1}-\left(r-r_p\right)^{\frac{1}{n}+1}\right] \quad (3.5)$$

This is the formula for velocity of blood flow in arterioles, veinules and veins.

Putting $r=r_p$ to get the velocity V_p of plug flow as follows:

$$V_p=\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1} \quad (3.6)$$

Where the value of r_p is taken from (2.7)

(4) Result (Bio-physical interpretation) –

Observations: Hematocrit Vs. Blood pressure from an authorized Jabalpur Hospital & Research Centre by Dr. Anil Jain

Patient Name: - Mr. Dinesh Singh

Diagnosis: - Diabetic/HT

Date	HB(Hemoglobin)	B.P.(blood Pressure) (Diastolic)	Hematocrit
31/7/11	4.2	0.675508 p.s	12.6
2/8/11	7.6	0.6755038 p.s	22.8
3/8/11	8.7	0.67550 38p.s	26.1
4/8/11	9.8	0.6004518 p.s.	29.4
6/8/11	12.0	0.6004518p.s	36.0

The flow flux of two phased blood flow in arterioles, veinules and veins is

$$Q=\int_0^{r_p}2\pi r V_p dr+\int_{r_p}^{R}2\pi r V dr$$

$$=\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1}dr+\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left[\left(R-r_p\right)^{\frac{1}{n}+1}-\left(r-r_p\right)^{\frac{1}{n}+1}\right]dr$$

Using (3.4) and (3.6)

$$=\frac{\pi n}{(n+1)}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3}\left[\frac{r_p^2}{R^2}\left(1-\frac{r_p^2}{R}\right)^{\frac{1}{n}+1}+\left(1+\frac{r_p}{R}\right)\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}-\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)}+\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]$$

(4.1)

Q=900 ml. /min = .015 litr /sec. R =1, $r_p = \frac{1}{3}$ H = 12.6According to Gustafson, Daniel R. (1980) p=0.6755 pascal-second

η_p = 0.0015 (Pascal-sec.) According to Glenn Elert (2010) η_m = 0.035 (pascal-sec.)

$$\eta_m = \eta_c X + \eta_p (1-X) \text{ where, } X = \frac{H}{100}$$

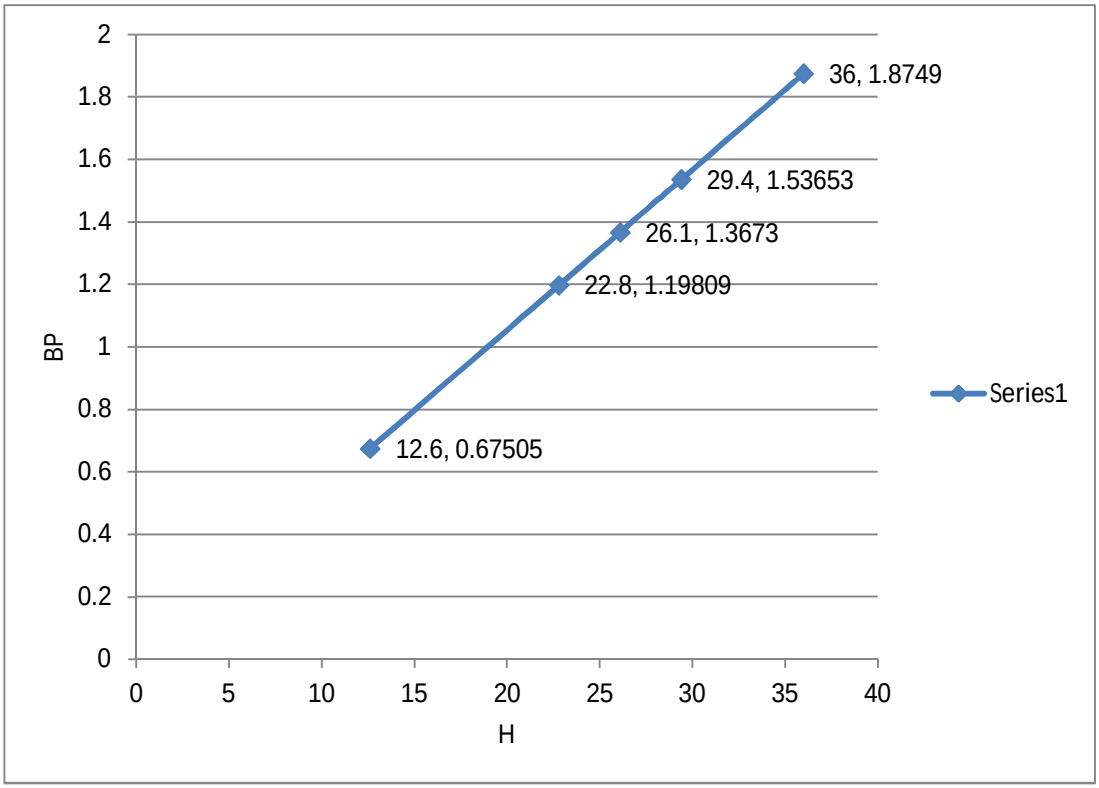
$$0.0644904 = (6.433)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$ Solved by Numerical method,

We get, n = -.86415

From equation (5.1)

$$\Rightarrow p = (.002658H + .0015)(.0644904)^{-.86415} \times \left[\frac{6n^3+11n^2+6n+1}{26n^3+33n^2+9n}\right]^n$$

H (Hematocrit)	12.6	22.6	26.1	29.4	36.0
P(Blood Pressure)	0.67505	1.19809	1.3673	1.53653	1.8749



Graph: (1)

(5) Conclusion:

A simple survey of the graph between blood pressure and hematocrit in Diabetic patient shows that when hematocrit increased then Blood pressure also increased. That is Hematocrit proportional to blood pressure.

Acknowledgement:

I owe my sincere thanks to Dr. Anil Jain, nephrologists of Jabalpur Hospital & Research Centre and thanks are due to Dr. Aashish V. Sharma , Administrator of Jabalpur Hospital & Research Centre.

Remark:

If this would have been possible to get blood Pressure on the particular tissue (Kidney) then the relation between blood pressure and hemoglobin has been measured more accurately.

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